

Exam 3 Formula Sheet

- **The Second Derivative Test for Functions of Two Variables**

Let $f(x, y)$ be a twice differentiable function, and assume its second partial derivatives are continuous. Let (a, b) be a critical point for f , and define the Hessian of f at (a, b) to be

$$D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2 .$$

- If $D(a, b) < 0$, then (a, b) is a Saddle Point.
- If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a Local Minimum.
- If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a Local Maximum.
- If $D(a, b) = 0$, the test is inconclusive.

- **Lagrange Multipliers**

If a differentiable function f has a local maximum or local minimum on a constraint of the form $g = \text{constant}$ (where g is a differentiable function), then there is a constant λ so that

$$\nabla f = \lambda \nabla g$$

at the local extremum (so long as ∇g is not zero at the point in question).

For multiple constraints, you add a separate multiplier for each. As an example, if f has a local maximum or minimum on the intersection of constraints $g = c_1$ and $h = c_2$ (where g and h are differentiable functions), then there are constants λ and μ so that

$$\nabla f = \lambda \nabla g + \mu \nabla h.$$